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IDENTIFICATION OF THE OIL-FILM DYNAMIC COEFFICIENTS IN A ROTOR-BEARING SYSTEM WITH A HYDRODYNAMIC THRUST BEARING

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1. INTRODUCTION

It has been revealed by recent rotordynamic studies that a hydrodynamic thrust bearing which is often treated as an axial support may show effect upon the lateral vibration of a rotor-bearing system. This effect was theoretically and experimentally discovered by Mittwollen *et al.* [1]. Jiang *et al.* have given an extensive theoretical investigation into the static and dynamic behavior of a rotor-bearing system equipped with a hydrodynamic thrust bearing [2, 3]. The effect is still attracting interests [4, 5]. The thrust bearing changes both the static and the dynamic boundary conditions of the system, and consequently influences the vibration modes, the static loads and dynamic coefficients of journal bearings. Therefore, the introduction of thrust bearing in theoretical analyses can explain some cases when the system stability and critical speeds deviate from those in conventional analyses where this effect is neglected. An experiment was conducted to reveal the effect of thrust bearing on system stability in reference [1]. To the authors' knowledge, there is no experiment on the identification of the oil-film dynamic coefficients of thrust bearings, which is the objective of the present paper.

There are lots of investigations into the identification of oil-film dynamic coefficients of journal bearings and squeeze-film dampers [6–9]. The system identification methods have been dominant in recent years, which can be classified into two categories, namely the time-domain methods [6] and the frequency-domain methods [7]. In the frequency-domain methods, the frequency response functions of system at various exciting frequencies are measured, from which the dynamic coefficients are estimated by using the least-squares method. Though exciters are needed, as they possess favorable anti-noise ability, they are still being extensively applied [9]. In this paper, the applicability of the frequency-domain methods in the identification of all the dynamic coefficients of thrust bearing and journal bearings is examined.

2. FORMULATIONS

A rotor-bearing system equipped with a hydrodynamic thrust bearing is shown in Figure 1. The equations of motion for the shaft can be formulated by using the finite-element method or the lumped-parameter method as

$$[M]\{\ddot{x}\} + [C]\{\ddot{x}\} + [K]\{x\} = \{f\}, \tag{1}$$

where $\{x\} \in R^{4N}$ consists of the generalized displacements including the deflection angles and displacements, N is the number of elements, $[M]$, $[C]$ and $[K]$ are the mass, damping

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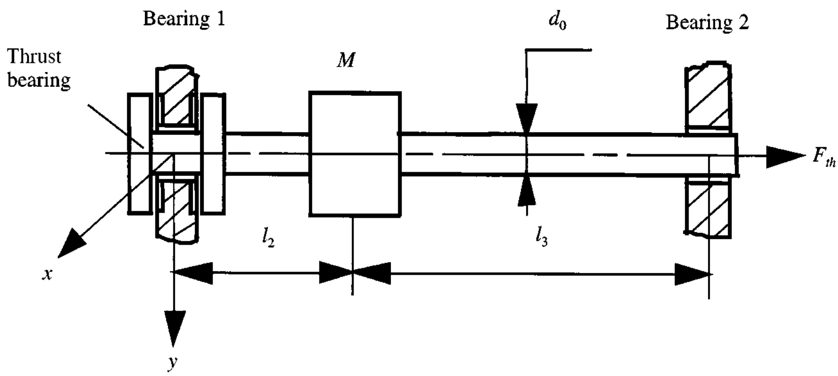

Figure 1. Schematic diagram of a rotor-bearing system with a hydrodynamic thrust bearing.

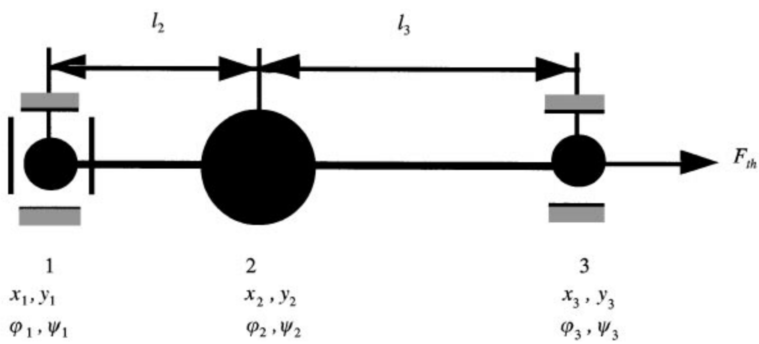

Figure 2. Lumped-parameter model of the rotor system.

and stiffness matrices of shaft, respectively, $\{f\} \in R^{4N}$ represents the external force including the excitations $\{f_{ex}\}$, the oil-film forces of journal bearings $\{f_{jb}\}$ and the oil-film forces and moments of thrust bearing $\{f_{tb}\}$, etc., i.e.,

$$\{f\} = \{f_{ex}\} + \{f_{jb}\} + \{f_{tb}\}, \tag{2}$$

where

$$\{f_{jb}\} = \begin{Bmatrix} \vdots \\ f_{jbxi} \\ f_{jbyi} \\ \vdots \end{Bmatrix}, \quad \{f_{tb}\} = \begin{Bmatrix} \vdots \\ f_{tbx} \\ f_{tby} \\ m_{tby} \\ m_{tbx} \\ \vdots \end{Bmatrix}.$$

On the assumption of small perturbation, the dynamic oil-film forces of journal bearings can be linearized into

$$\begin{Bmatrix} f_{jbx} \\ f_{jby} \end{Bmatrix}_i = -\begin{bmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{bmatrix}_i \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix}_i - \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_i \begin{Bmatrix} x \\ y \end{Bmatrix}_i$$

$$(i = 1, \ldots, N_b.\ N_b \text{ is the number of journal bearings}). \tag{3}$$

In the present paper, we suppose that there is no axial load, and the thrust bearing is a double-facet and self-balance one. Therefore, the dynamic forces are 0, and dynamic moments of thrust bearing can be written as [3]

$$\begin{Bmatrix} m_{tby} \\ m_{tbx} \end{Bmatrix} = -\begin{bmatrix} d^M_{y\varphi} & d^M_{y\psi} \\ -d^M_{x\varphi} & -d^M_{x\psi} \end{bmatrix} \begin{Bmatrix} \dot{\varphi} \\ \dot{\psi} \end{Bmatrix} - \begin{bmatrix} k^M_{y\varphi} & k^M_{y\psi} \\ -k^M_{x\varphi} & -k^M_{x\psi} \end{bmatrix} \begin{Bmatrix} \varphi \\ \psi \end{Bmatrix}. \quad (4)$$

Substitution of equation (4) into equation (1) gives

$$[M]\{\ddot{x}\} + [\tilde{C}]\{\dot{x}\} + [\tilde{K}]\{x\} = \{f_{im}\}, \quad (5)$$

the dynamic coefficients of journal bearings and thrust bearing are included in matrices $[\tilde{K}]$ and $[\tilde{C}]$.

In the frequency domain, equation (5) is in the form

$$[A(\mathrm{j}\omega_k)]\{X(\mathrm{j}\omega_k)\} = \{F_{im}(\mathrm{j}\omega_k)\}, \quad (6)$$

where $[A(\mathrm{j}\omega_k)] = -\omega_k^2[M] + \mathrm{j}\omega_k[\tilde{C}] + [\tilde{K}]$, $\mathrm{j} = \sqrt{-1}$, ω_k is the frequency of the kth harmonic components, $k = 1, \ldots, m$.

In practice, usually the deflection angles are not easy to measure, and only the displacements of shaft at the journal bearings are measured, therefore equation (6) is partitioned as

$$\begin{bmatrix} A_{11}(\mathrm{j}\omega_k) & A_{12}(\mathrm{j}\omega_k) & A_{13}(\mathrm{j}\omega_k) \\ A_{21}(\mathrm{j}\omega_k) & A_{22}(\mathrm{j}\omega_k) & A_{23}(\mathrm{j}\omega_k) \\ A_{31}(\mathrm{j}\omega_k) & A_{32}(\mathrm{j}\omega_k) & A_{33}(\mathrm{j}\omega_k) \end{bmatrix} \begin{Bmatrix} X_1(\mathrm{j}\omega_k) \\ X_2(\mathrm{j}\omega_k) \\ X_3(\mathrm{j}\omega_k) \end{Bmatrix} = \begin{Bmatrix} F_{im1}(\mathrm{j}\omega_k) \\ F_{im2}(\mathrm{j}\omega_k) \\ F_{im3}(\mathrm{j}\omega_k) \end{Bmatrix}, \quad (7)$$

where $X_2(\mathrm{j}\omega_k) \in C^{2N_b}$ consists of the displacements where the journal bearings act, $X_3(\mathrm{j}\omega_k) \in C^2$ consists of the angles where the thrust bearing acts, $X_1(\mathrm{j}\omega_k) \in C^{4N-2N_b}$ consists of the other displacements and angles. Without loss of generality, we suppose that the thrust bearing is combined with a journal bearing. Therefore in equation (7), $[A_{22}(\mathrm{j}\omega_k)]$ includes the oil-film dynamic coefficients of journal bearings, and $[A_{33}(\mathrm{j}\omega_k)]$ includes the moment dynamic coefficients of thrust bearing. The other matrices are obtained from the finite element model of the rotor, and are assumed to be sufficiently accurate.

In equation (7), $\{X_1(\mathrm{j}\omega_k)\}$ and $\{X_3(\mathrm{j}\omega_k)\}$ are intermediate unknowns, and all the dynamic coefficients are the unknowns to be identified. From equation (7), we have

$$E_{1k} = [A_{22}(\mathrm{j}\omega_k)]\{X_2(\mathrm{j}\omega_k)\} + [A_{21}(\mathrm{j}\omega_k)][A_{11}(\mathrm{j}\omega_k)]^{-1}(\{F_{im1}(\mathrm{j}\omega_k)\} - [A_{12}(\mathrm{j}\omega_k)]\{X_2(\mathrm{j}\omega_k)\}$$
$$- [A_{13}(\mathrm{j}\omega_k)]\{X_3(\mathrm{j}\omega_k)\}) + [A_{23}(\mathrm{j}\omega_k)]\{X_3(\mathrm{j}\omega_k)\} - \{F_{im2}(\mathrm{j}\omega_k)\} = 0, \quad (8)$$

$$E_{2k} = [A_{33}(\mathrm{j}\omega_k)]\{X_3(\mathrm{j}\omega_k)\} + [A_{31}(\mathrm{j}\omega_k)][A_{11}(\mathrm{j}\omega_k)]^{-1}(\{F_{im1}(\mathrm{j}\omega_k)\} - [A_{12}(\mathrm{j}\omega_k)]\{X_2(\mathrm{j}\omega_k)\}$$
$$- [A_{13}(\mathrm{j}\omega_k)]\{X_3(\mathrm{j}\omega_k)\}) + [A_{32}(\mathrm{j}\omega_k)]\{X_2(\mathrm{j}\omega_k)\} - \{F_{im3}(\mathrm{j}\omega_k)\} = 0. \quad (9)$$

Take a single-disk rotor laterally supported by two journal bearings (Figure 1) for example. If the shaft is discretized into three elements by using the lumped-parameter method, the matrices and vectors in equations (8) and (9) are

$$[A_{11}] = \begin{bmatrix} \frac{12EI_2}{l_2^3} + \frac{12EI_3}{l_3^3} - m_2\omega_k^2 & 0 & \frac{6EI_3}{l_3^2} - \frac{6EI_2}{l_2^2} & 0 & \frac{6EI_3}{l_3^2} & 0 \\ 0 & \frac{12EI_2}{l_2^3} + \frac{12EI_3}{l_3^3} - m_2\omega_k^2 & 0 & \frac{6EI_3}{l_3^2} - \frac{6EI_2}{l_2^2} & 0 & \frac{6EI_3}{l_3^2} \\ \frac{6EI_3}{l_3^2} - \frac{6EI_2}{l_2^2} & -\mathrm{j}\omega_k^2\theta_{z2} & \frac{4EI_3}{l_3} + \frac{4EI_2}{l_2} - \theta_{x2}\omega_k^2 & 0 & \frac{2EI_3}{l_3} & 0 \\ \mathrm{j}\omega_k^2\theta_{z2} & \frac{6EI_3}{l_3^2} - \frac{6EI_2}{l_2^2} & 0 & \frac{4EI_3}{l_3} + \frac{4EI_2}{l_2} - \theta_{x2}\omega_k^2 & 0 & \frac{2EI_3}{l_3} \\ \frac{6EI_3}{l_3^2} & 0 & \frac{2EI_3}{l_3} & 0 & \frac{4EI_3}{l_3} - \theta_{x2}\omega_k^2 & 0 \\ 0 & \frac{6EI_3}{l_3^2} & 0 & \frac{2EI_3}{l_3} & 0 & \frac{4EI_3}{l_3} - \theta_{x2}\omega_k^2 \end{bmatrix},$$

$$[A_{22}] = \begin{bmatrix} \frac{12EI_2}{l_2^3} + k_{xx1} + \mathrm{j}\omega_k d_{xx1} - m_1\omega_k^2 & k_{xy1} + \mathrm{j}\omega_k d_{xy1} & 0 & 0 \\ k_{yx1} + \mathrm{j}\omega_k d_{yx1} & \frac{12EI_2}{l_2^3} + k_{yy1} + \mathrm{j}\omega_k d_{yy1} - m_1\omega_k^2 & 0 & 0 \\ 0 & 0 & \frac{12EI_3}{l_3^3} + k_{xx2} + \mathrm{j}\omega_k d_{xx2} - m_3\omega_k^2 & k_{xy2} + \mathrm{j}\omega_k d_{xy2} \\ 0 & 0 & k_{yx2} + \mathrm{j}\omega_k d_{yx2} & \frac{12EI_3}{l_3^3} + k_{yy2} + \mathrm{j}\omega_k d_{yy2} - m_3\omega_k^2 \end{bmatrix},$$

$$[A_{33}] = \begin{bmatrix} \dfrac{4EI_2}{l_2} + k^M_{y\varphi} + \mathrm{j}\omega_k d^M_{y\varphi} - \theta_{y1}\omega_k^2 & k^M_{y\psi} + \mathrm{j}\omega_k d^M_{y\psi} \\ k^M_{x\varphi} + \mathrm{j}\omega_k d^M_{x\varphi} & \dfrac{4EI_2}{l_2} + k^M_{x\psi} + \mathrm{j}\omega_k d^M_{x\psi} - \theta_{x1}\omega_k^2 \end{bmatrix},$$

$$[A_{12}] = \begin{bmatrix} -\dfrac{12EI_2}{l_2^3} & 0 & -\dfrac{12EI_3}{l_3^3} & 0 \\ 0 & -\dfrac{12EI_2}{l_2^3} & 0 & -\dfrac{12EI_3}{l_3^3} \\ \dfrac{6EI_2}{l_2^2} & 0 & -\dfrac{6EI_3}{l_3^2} & 0 \\ 0 & \dfrac{6EI_2}{l_2^2} & 0 & -\dfrac{6EI_3}{l_3^2} \\ 0 & 0 & -\dfrac{6EI_3}{l_3^2} & 0 \\ 0 & 0 & 0 & -\dfrac{6EI_3}{l_3^2} \end{bmatrix}, \quad [A_{13}] = \begin{bmatrix} -\dfrac{6EI_2}{l_2^2} & 0 \\ 0 & -\dfrac{6EI_2}{l_2^2} \\ \dfrac{2EI_2}{l_2} & 0 \\ 0 & \dfrac{2EI_2}{l_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$[A_{21}] = \begin{bmatrix} -\dfrac{12EI_2}{l_2^3} & 0 & \dfrac{6EI_2}{l_2^2} & 0 & 0 & 0 \\ 0 & -\dfrac{12EI_2}{l_2^3} & 0 & \dfrac{6EI_2}{l_2^2} & 0 & 0 \\ -\dfrac{12EI_3}{l_3^3} & 0 & -\dfrac{6EI_3}{l_3^2} & 0 & -\dfrac{6EI_3}{l_3^2} & 0 \\ 0 & -\dfrac{12EI_3}{l_3^3} & 0 & -\dfrac{6EI_3}{l_3^2} & 0 & -\dfrac{6EI_3}{l_3^2} \end{bmatrix},$$

$$[A_{13}] = \begin{bmatrix} -\dfrac{6EI_2}{l_2^2} & 0 \\ 0 & -\dfrac{6EI_2}{l_2^2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$[A_{31}] = \begin{bmatrix} -\dfrac{6EI_2}{l_2^2} & 0 & \dfrac{2EI_2}{l_2} & 0 & 0 & -\mathrm{j}\omega_k^2\theta_{z3} \\ 0 & \dfrac{6EI_2}{l_2^2} & 0 & \dfrac{2EI_2}{l_2} & \mathrm{j}\omega_k^2\theta_{z3} & 0 \end{bmatrix},$$

$$[A_{32}] = \begin{bmatrix} -\dfrac{6EI_2}{l_2^2} & -\mathrm{j}\omega_k^2\theta_{z1} & 0 & 0 \\ \mathrm{j}\omega_k^2\theta_{z1} & -\dfrac{6EI_2}{l_2^2} & 0 & 0 \end{bmatrix},$$

$$\{X_1\} = \{X_{20}, Y_{20}, \Phi_{20}, \Psi_{20}, \Phi_{30}, \Psi_{30}\}^{\mathrm{T}}, \qquad \{X_2\} = \{X_{10}, Y_{10}, X_{30}, Y_{30}\}^{\mathrm{T}},$$

$$\{X_3\} = \{\Phi_{10}, \Psi_{10}\}^{\mathrm{T}},$$

where E is elastic module, θ_x, θ_y and θ_z are moments of inertia, $I = \pi d^4/64$, and d is the diameter of the shaft.

It is obvious that equation (9) is non-linear, which differs from that for a system solely supported by journal bearings [7] where the identification equations are linear ones, therefore the least-squares method for linear identification equations cannot be applied directly. Application of non-linear programming methods such as the Broyden-Fletcher–Goldfarb–Shanno (BFGS) method may be necessary. Therefore, the problem is transformed into a minimization problem, i.e.,

$$\min \sum_{k=1}^{m} (|E_{1k}|^2 + |E_{2k}|^2). \tag{10}$$

The choice of the initial values of unknowns is of great importance for the BFGS method. In this paper, a hierarchical procedure is presented to give the guess of initial values, which is described as follows:

(1) As equation (8) is linear, the dynamic coefficients of journal bearings and $\{X_3(\mathrm{j}\omega_k)\}$ can be estimated by using the least-squares method.
(2) Substituting $\{X_3(\mathrm{j}\omega_k)\}$ into equation (9), the latter becomes linear. The dynamic coefficients of thrust bearing can be obtained thereafter by using the least-squares method.

In fact, the hierarchical procedure itself can estimate all the dynamic coefficients. But the combination of it with the BFGS method can give higher fitting precision.

3. NUMERICAL SIMULATION

Numerical simulation on a single-disk rotor-bearing system as shown in Figure 1 is performed to examine the applicability of the method described above. The rotor is supported by two identical 360° cylindrical journal bearings at both ends. A double-facet fixed-pad thrust bearing is attached at the left end, and is integrated with the left journal bearing to form a combined bearing.

The parameters of the rotor are: diameter $d_0 = 50$ mm, $l_2 = 20$ mm, $l_3 = 60$ mm, mass of disk $M = 75$ kg, rotational inertia of disk $\theta_x = \theta_y = 0{\cdot}75$ kg m^2, $\theta_z = 1{\cdot}5$ kg m^2.

The parameters of the journal bearings are: diameter $D_0 = 50$ mm, ratio of length to diameter $L/D_0 = 0{\cdot}5$, clearance ratio $\Psi = 0{\cdot}001$. The parameters of the thrust bearings are: width of pad $B = 50$ mm, film thickness on pitch line $h_p = 0{\cdot}05$ mm, angular extent of pad $\theta_0 = 40°$, angular position of pitch line $\theta_p = \theta_0/2$, inner radius $r_1 = 50$ mm, wedge angle of pad $\alpha_0 = 0{\cdot}002$ rad, reference film thickness $h_e = 0{\cdot}05$ mm. The dynamic viscosity of oil $\mu = 0{\cdot}027$ N s/m^2, the axial load $F_{thb} = 0{\cdot}0$, and the rotating speed $\Omega = 4500$ r/min.

All the bearings are assumed to work in the isothermal laminar condition. The static forces and moments and dynamic coefficients of the thrust bearing are calculated from the *Reynolds* equation and its perturbed forms by using the boundary element method. The short-bearing model is used to calculate the static forces and dynamic coefficients of the journal bearings. An impulse of 20 ms width, which is used to simulate the impact force, is

TABLE 1

Calculated and identified results

	$k^M_{x\varphi}/c_1$	$k^M_{x\psi}/c_1$	$d^M_{x\varphi}/c_2$	$d^M_{x\psi}/c_2$	$k^M_{y\varphi}/c_1$	$k^M_{y\psi}/c_1$	$d^M_{y\varphi}/c_2$	$d^M_{y\psi}/c_2$	$k^{(1)}_{xx}/c_3$	$k^{(1)}_{xy}/c_3$	$k^{(1)}_{yx}/c_3$	$k^{(1)}_{yy}/c_3$
Computed results	0·6188	3·2926	0·0058	1·2488	− 3·2870	0·6175	− 1·2477	0·0054	0·2331	−0·7837	0·8507	0·1247
Estimated results with no noise	0·6202	3·2948	0·0056	1·2441	− 3·2906	0·6182	− 1·2432	0·0051	0·2349	− 0·7855	0·8527	0·1265
Estimated results with 40% noises in both input and output signals	0·6167	3·3188	0·0054	1·2436	− 3·3175	0·6224	− 1·2413	0·0056	0·2325	− 0·7929	0·8425	0·1226
	$d^{(1)}_{xx}/c_4$	$d^{(1)}_{xy}/c_4$	$d^{(1)}_{yx}/c_4$	$d^{(1)}_{yy}/c_4$	$k^{(2)}_{xx}/c_3$	$k^{(2)}_{xy}/c_3$	$k^{(2)}_{yx}/c_3$	$k^{(2)}_{yy}/c_3$	$d^{(2)}_{xx}/c_4$	$d^{(2)}_{xy}/c_4$	$d^{(2)}_{yx}/c_4$	$d^{(2)}_{yy}/c_4$
Computed results	1·5961	0·2333	0·2333	1·6727	0·0380	− 0·7857	0·7874	0·0190	1·5729	0·0380	0·0380	1·5732
Estimated results with no noise	1·6021	0·2321	0·2321	1·6815	0·0392	− 0·7866	0·7889	0·0195	1·5820	0·0366	0·0366	1·5821
Estimated results with 40% noises in both input and output signals	1·6124	0·2341	0·2302	1·6547	0·0386	− 0·7903	0·7927	0·0181	1·5916	0·0360	0·0354	1·5902

Note: $c_1 = \mu\omega B^6/h_e^3$, $c_2 = \mu B^6/h_e^3$, $c_3 = \mu\omega L/\Psi^3$, $c_4 = \mu L/\Psi^3$.

exerted at the lumped mass M. The system responses are calculated by using the fourth order *Runge–Kutta* method.

The results from numerical calculation and identification are given in Table 1. A comparison between case(a) and case(b) shows that the method can succeed in identifying all the dynamic coefficients of journal bearings and thrust bearing.

The effect of noise is also discussed. The noise level NL is defined as

$$NL = SD_{noise}/SD_{signal}, \tag{11}$$

where SD means standard deviation.

Case(c) in Table 1 gives the estimated results when 40% additive *Gaussian* noises are added to both the input and output signals. It is shown that the frequency method has favorable anti-noise ability.

4. SUMMARY

The identification equations for a rotor-bearing system equipped with a hydrodynamic bearing are formulated. The angles where the thrust bearing acts need not be measured. The BFGS method combined with a hierarchical procedure is used to solve the identification equations. The applicability of the frequency-domain method in the identification of all the oil-film dynamic coefficients of journal bearings and thrust bearing is confirmed by numerical simulation. It is also shown that the method is of high anti-noise ability.

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